Unit - I

Unsymmetrical Bending

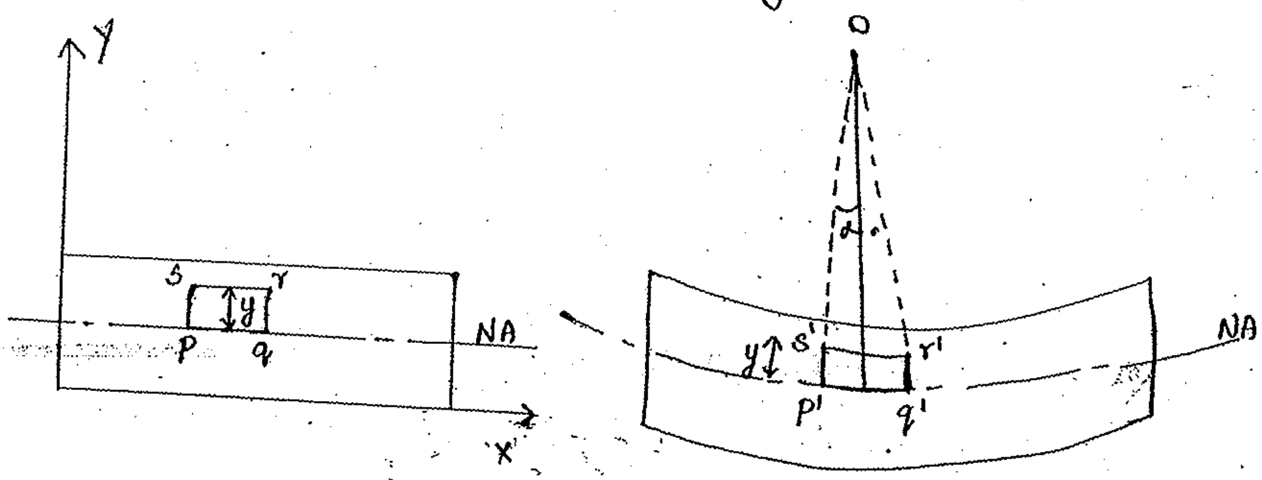
Consider the beam Subjected to ~~beam~~ bending due to lateral loads and bending moment, if the beam bends in the plane of loads, then it is Said to be Symmetrical bending.

If the beam bends not only in the plane of loads but also Perpendicular to the load, then it is Said to be Unsymmetrical bending.

Bending Equation.

Assumptions made in theory of Simple bending:-

1. Material is Homogenous.
2. Material of beam is uniform throughout.
3. Each cross Section of beam is Symmetric about the plane of bending.
4. The loads are applied to the beam in the plane of bending
5. Youngs Modulus have Same Value in compression & Tension.
6. Hookes law applied to each longitudinal layers.



Consider the beam Subjected to bending. Figure Shows the longitudinal Section of neutral axis we bend to form an arc of a circle of Radius "R". The Neutral layer is pq before bending, p'q' after bending.

Consider a layer rs at a distance y from pq which becomes r's' after bending. Let p'q' have Some angle of α from centre of Curvature O.

$$p'q' = R\alpha$$
$$r's' = (R-y)\alpha$$

At Neutral axis, $pq = p'q' = rs$.

$$\text{Strain } \varepsilon = \frac{\text{Change in length}}{\text{Original length}} = \frac{rs - r's'}{rs}$$

$$= \frac{R\alpha - (R-y)\alpha}{R\alpha} = y/R \rightarrow (1)$$

We know that Youngs Modulus $E = \frac{\sigma}{\varepsilon}$

$$\varepsilon = \frac{\sigma}{E} = y/R \quad (\text{from } (1))$$

$$\therefore \frac{\sigma}{y} = \frac{E}{R} \Rightarrow \sigma = \frac{E}{R} y$$

Consider a transverse Section of a beam, let a Small elemental area dA lie at a distance y from Neutral axis

$$\partial F = \sigma \partial A.$$
$$\partial M = \partial F \cdot y$$

$$= \sigma \, \partial A \cdot y$$

$$M = \int \sigma \cdot y \cdot dA$$

$$= \int \frac{E}{R} y^2 \, dA \qquad \because \sigma = \frac{E}{R} y$$

$$M = \frac{E}{R} I$$

$$\boxed{\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}}$$

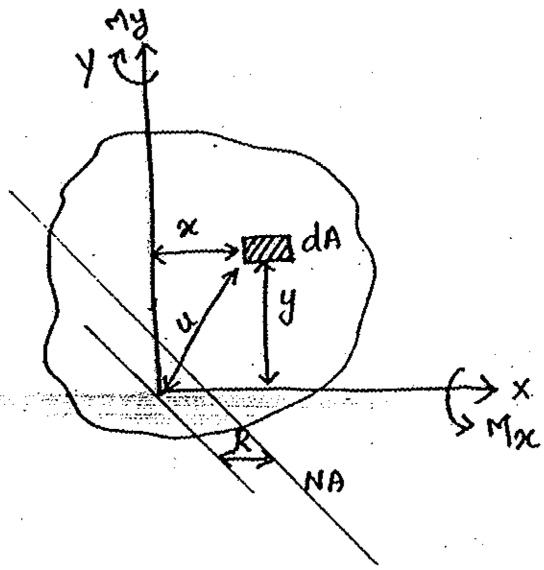
This equation is said to be Bending equation.

Usymmetric Bending equation. (Bending Stress method)

Incase of unsymmetrical bending the direction of neutral axis is not perpendicular to plane of bending because

→ The Section is Symmetrical, but load line is inclined to both the principle axis.

→ The Section itself is unsymmetrical.



$$\sigma_z = E\varepsilon$$

$$\varepsilon = \frac{U}{R} \text{ (From fig)}$$

$$\boxed{\sigma_z = \frac{EU}{R}} \rightarrow ①$$

For pure bending,

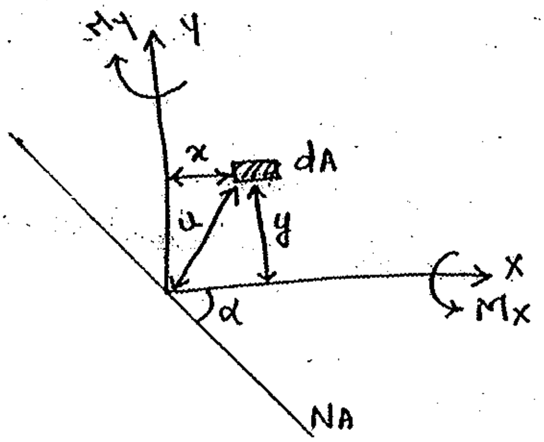
$$\int \sigma \, dA = 0$$

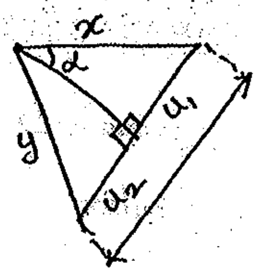
$$\int \frac{EU}{R} dA = 0$$

$$\frac{E}{R}\int U \, dA = 0$$

$$\Rightarrow \int U \, dA = 0 \longrightarrow (2) \quad \because \frac{E}{R} \neq 0$$

This equation states that first moment of area of ~~moment~~ beam section about the neutral axis is zero. It follows that the neutral axis always pass through the centroid of the beam.





$$\sin\alpha = \frac{u_1}{x}$$

$$u_1 = x\sin\alpha$$

$$\cos\alpha = \frac{u_2}{y}$$

$$u_2 = y\cos\alpha$$

$$U = u_1 + u_2 = x\sin\alpha + y\cos\alpha$$

Equation (1) becomes

$$\sigma_z = \frac{E}{R} U$$

$$\sigma_z = \frac{E}{R}\left[x\sin\alpha + y\cos\alpha\right] \longrightarrow (A)$$

$$\partial F = \sigma_z \, dA$$

$$\partial M_x = \sigma_z \, dA \cdot y$$

$$= \frac{E}{R} (x \sin\alpha + y \cos\alpha) \, dA \cdot y$$

$$M_x = \int \frac{E}{R} (x \sin\alpha + y \cos\alpha) \, dA \cdot y$$

$$= \frac{E}{R} \int (xy \sin\alpha \, dA + y^2 \cos\alpha \, dA)$$

We know that,

$$I_{xy} = \int xy \, dA$$

$$I_{xx} = \int y^2 \, dA$$

$$I_{yy} = \int x^2 \, dA.$$

$$M_x = \frac{E}{R} \int \left[I_{xy} \sin\alpha + I_{xx} \cos\alpha \right] \longrightarrow (3)$$

$$M_y = \frac{E}{R} \int \left[I_{yy} \sin\alpha + I_{xy} \cos\alpha \right] \longrightarrow (4)$$

Multiply equation (3) by I_{yy} and Equation (4) by I_{xy}

$$M_x I_{yy} = \frac{E}{R} \int \left[I_{xy} \sin\alpha \, I_{yy} + I_{xx} I_{yy} \cos\alpha) \right]$$

$$M_y I_{xy} = \frac{E}{R} \int \left[I_{yy} I_{xy} \sin\alpha + I_{xy}^2 \cos\alpha \right]$$

$$M_x I_{yy} - M_y I_{xy} = \frac{E}{R} \left[I_{xx} I_{yy} - I_{xy}^2 \right] \cos\alpha.$$

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$$\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} = \frac{E}{R} \cos\alpha \rightarrow (5)$$

Multiply equation (3) by I_{xy} and equation (4) by I_{xx}.

$$\Rightarrow M_x \cdot I_{xy} = \frac{E}{R} \int \left[I_{xy}^2 \sin\alpha + I_{xx} I_{xy} \cos\alpha \right]$$

$$M_y \cdot I_{xx} = \frac{E}{R} \int \left[I_{xx} I_{yy} \sin\alpha + I_{xx} I_{xy} \cos\alpha \right]$$

$$\left[M_x I_{xy} - M_y I_{xx} \right] = \frac{E}{R} \left[I_{xy}^2 - I_{xx} I_{yy} \right] \sin\alpha$$

$$- \frac{M_x I_{xy} - M_y I_{xx}}{I_{xy}^2 - I_{xx} I_{yy}} = \frac{E}{R} \sin\alpha$$

$$\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} = \frac{E}{R} \sin\alpha \rightarrow (6)$$

Substitute equation (5) & (6) in A.

$$\sigma_z = \left(\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \right) x + \left(\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \right) y$$

$$\sigma_z = \frac{I_{yy}}{I_{xx} I_{yy}} \left[\frac{M_x - M_y \frac{I_{xy}}{I_{yy}}}{1 - \frac{I_{xy}^2}{I_{xx} I_{yy}}} \right] y + \frac{I_{xx}}{I_{xx} I_{yy}} \left[\frac{M_y - M_x \frac{I_{xy}}{I_{xx}}}{1 - \frac{I_{xy}^2}{I_{xx} I_{yy}}} \right] x$$

$$\sigma_z = \frac{\overline{M}_x}{I_{xx}} \cdot y + \frac{\overline{M}_y}{I_{yy}} x.$$

where.

$$\overline{M}_x = \frac{M_x - M_y \frac{I_{xy}}{I_{yy}}}{1 - \frac{I_{xy}^2}{I_{xx} I_{yy}}} \quad ; \quad \overline{M}_y = \frac{M_y - M_x \frac{I_{xy}}{I_{xx}}}{1 - \frac{I_{xy}^2}{I_{xx} I_{yy}}}$$

In case of Symmetrical Section with any one of axis, the Product of moment of Inertia is zero.

$$I_{xy} = 0.$$

(1) Principal Axis Method :-

If the axis are Principle axis,

$$\sigma_z = \frac{M_{uu}}{I_{uu}} u + \frac{M_{vv}}{I_{vv}} v$$

where $u = y\sin\theta + x\sin\theta.$

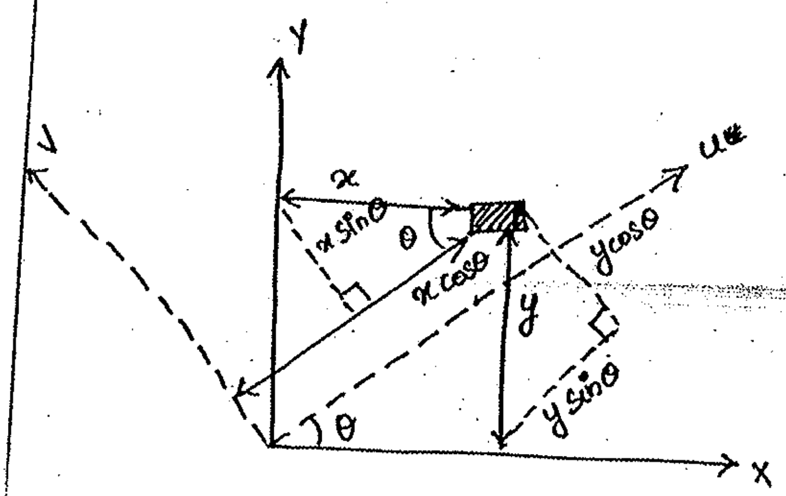
$v = y\cos\theta - x\sin\theta$

(or)

$$\boxed{\sigma_z = \frac{M_{xp}}{I_{xp}} y_p + \frac{M_{yp}}{I_{yp}} x_p}$$

$x_p = x\cos\theta + y\sin\theta$

$y_p = y\cos\theta \mp x\sin\theta$



$$I_{xp} = \int y^2 dA$$

$$= \int [y\cos\theta - x\sin\theta]^2 dA$$

$$= \int [y^2\cos^2\theta + x^2\sin^2\theta - 2xy\cos\theta\sin\theta]\, dA$$

$$= \int y^2\cos^2\theta\, dA + \int x^2\sin^2\theta\, dA - 2\int xy\cos\theta\sin\theta\, dA.$$

$$I_{xp} = I_{xx}\cos^2\theta + I_{yy}\sin^2\theta - 2I_{xy}\cos\theta\sin\theta$$

llely $I_{yp} = \int x_p^2\, dA.$

$$= \int x^2\cos^2\theta\, dA + \int y^2\sin^2\theta\, dA + 2\int xy\cos\theta\sin\theta\, dA$$

$$I_{yp} = I_{yy}\cos^2\theta + I_{xx}\sin^2\theta + 2I_{xy}\cos\theta\sin\theta.$$

$$I_{xp} + I_{yp} = I_{xx}(\cos^2\theta + \sin^2\theta) + I_{yy}(\cos^2\theta + \sin^2\theta)$$

$$I_{xp} + I_{yp} = I_{xx} + I_{yy}$$

$$I_{xyp} = \int x_p y_p\, dA.$$

$$= \int [xy\cos^2\theta - x^2\cos\theta\sin\theta + y^2\cos\theta\sin\theta - xy\sin^2\theta]\, dA.$$

$$= I_{xy}\cos^2\theta - I_{yy}\cos\theta\sin\theta + I_{xx}\cos\theta\sin\theta - I_{xy}\sin^2\theta.$$

$$I_{xyp} = I_{xy}(\cos^2\theta - \sin^2\theta) + \sin\theta\cos\theta\,(I_{xx} - I_{yy})$$

In Principle axis, $I_{xpyp} = 0.$

$$I_{xy}\cos 2\theta + \frac{\sin 2\theta}{2}\left[I_{xx} - I_{yy}\right] = 0$$

$$\frac{\sin 2\theta}{2}\left(I_{xx} - I_{yy}\right) = -I_{xy}\cos 2\theta$$

$$\tan 2\theta = \frac{-2I_{xy}}{I_{xx} - I_{yy}}$$

$$\tan 2\theta = \frac{2I_{xy}}{I_{yy} - I_{x}}$$

(ii) Neutral axis Method :

We know that,

$$\sigma_z = \frac{\bar{M}_x}{I_{xx}} y + \frac{\bar{M}_y}{I_{yy}} x$$

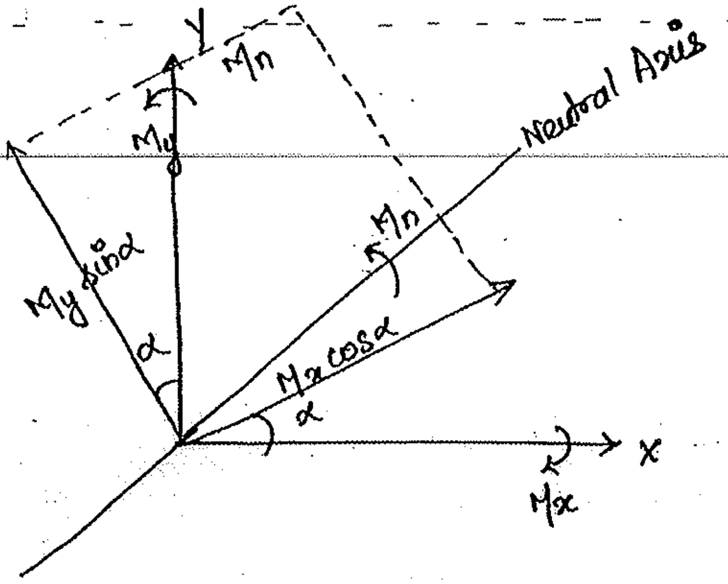
In neutral axis Method $\sigma_z = 0$

$$\frac{\bar{M}_x}{I_{xx}} y = -\frac{\bar{M}_y}{I_{yy}} x$$

$$\frac{y}{x} = -\frac{\bar{M}_y}{I_{yy}} \times \frac{I_{xx}}{\bar{M}_x}$$

$$\tan\alpha = -\frac{\bar{M}_y}{I_{yy}} \cdot \frac{I_{xx}}{\bar{M}_x}$$

$$\alpha = \tan^{-1}\left[-\frac{\bar{M}_y}{\bar{M}_x} \cdot \frac{I_{xx}}{I_{yy}}\right]$$



From the diagram,

$$M_n = M_x \cos\alpha - M_y \sin\alpha$$

$$y_n = y\cos\alpha - x\sin\alpha$$

$$I_n = \int y_n^2 \, dA.$$

$$= \int [y\cos\alpha - x\sin\alpha]^2 \, dA.$$

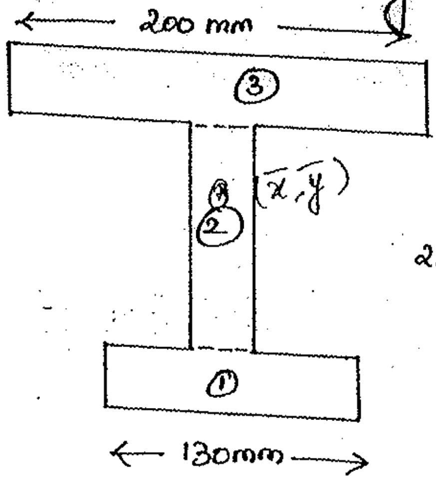
$$I_n = I_{xx}\cos^2\alpha + I_{yy}\sin^2\alpha - I_{xy}\sin 2\alpha$$

For neutral axis method, σ_z can be written as

$$\boxed{\sigma_z = \frac{M_n}{I_n} y_n}$$

Problem - 1

A cast iron Subjected to bending has cross section of unequal flange. The dimension of the reactions are shown in fig. Find the (i) position of Neutral axis (ii) Moment of Inertia of the Section at Neutral axis and also find (iii) Maximum bending stress. where thickness is 50mm & Bending moment is 40 MN-mm.



Data given

$t = 50\ mm$

$BM = 40\ MN\text{-}mm$

Solution:

$A_1 = 130 \times 50 = 6500\ mm^2$

$y_1 = \frac{50}{2} = 25\ mm$

$A_2 = 200 \times 50 = 10000\ mm^4$

$y_2 = 50 + \frac{200}{2}$

$= 150\ mm$

$A_3 = 200 \times 50$

$= 10000\ mm^4$

$y_3 = 250 + \frac{50}{2}$

$= 275\ mm$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{(6500 \times 25) + (10000 \times 150) + (10000 \times 275)}{6500 + 10000 + 10000}$$

$$\boxed{\bar{y} = 166.5\ mm}$$

$I_{xx} = I_1 + I_2 + I_3$.

$$I_1 = \frac{b_1 d_1^3}{12} + A_1 y_1^2 \qquad y_1 = (\bar{y} - y_1)$$

$$= \frac{130 \times 50^3}{12} + 6500 \times (166.5 - 25)^2$$

$I_1 = 1.3149 \times 10^8\ mm^4$.

IIIly $$I_2 = \frac{b_2 d_2^3}{12} + A_2 y_2^2 = \frac{50 \times 200^3}{12} + 10000\,(166.5 - 150)^2$$

$I_2 = 3.6055 \times 10^7\ mm^4$

$$I_3 = \frac{b_3 d_3^3}{12} + A_3 y_3^2$$

$$= \frac{200 \times 50^3}{12} + 10000\,(166.5 - 275)^2$$

$I_3 = 1.198 \times 10^8\ mm^4$

$I_{xx} = I_1 + I_2 + I_3$.

$= 1.3149 \times 10^8 + 3.6055 \times 10^7 + 1.198 \times 10^8$

$$\boxed{I_{xx} = 2.87 \times 10^8\ mm^4}$$

The Neutral axis is y. Here we find I_{xx}. (Neutral axis is ⊥er to axis of Symmetry always)

Maximum Bending Stress :-

$$\frac{M}{I} = \frac{\sigma}{y} \qquad (\because \bar{y} = y)$$

$$\sigma = \frac{M}{I} \cdot y = \frac{40 \times 166.5 \times 10^6}{2.87 \times 10^8} = 23.1772\ N/mm^2$$

Maximum Bending Stress,

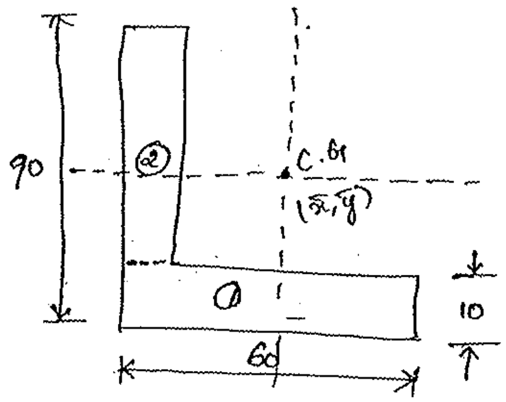
$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{M}{I} \times y = \frac{3.4 \times 10^9 \times 125}{4.9526 \times 10^7}$$

$$\boxed{\sigma = 8.581\ KN/mm^2}$$

Problem - 7

An unequal angle section $90 \times 60 \times 10$ is shown in figure. Find its I_{xx}, I_{yy} and I_{xy}.



$A_1 = 60 \times 10 = 600\ mm^2$ $\qquad$ $A_2 = 80 \times 10 = 800\ mm^2$

$y_1 = \frac{10}{2} = 5\ mm$ $\qquad$ $y_2 = 10 + \frac{80}{2} = 50\ mm$

$x_1 = \frac{60}{2} = 30\ mm$ $\qquad$ $x_2 = \frac{10}{2} = 5\ mm$

$\bar{x} = 15.71\ mm$ $\qquad$ $\bar{y} = 30.72\ mm$

$$I_{xx} = \frac{b_1 d_1^3}{12} + A_1 y_1^2 + \frac{b_2 d_2^3}{12} + A_2 y_2^2$$

$$= \frac{60 \times 10^3}{12} + 600(30.72 - 5)^2 + \frac{10 \times 80^3}{12} + 600(30.72 - 50)^2$$

$$\boxed{I_{xx} = 1.1259 \times 10^6\ mm^4.}$$

$$I_{yy} = \frac{d_1 b_1^3}{12} + A_1(\bar{x}-x_1)^2 + \frac{d_2 b_2^3}{12} + A_2(\bar{x}-x_2)^2$$

$$= \frac{10\times60^3}{12} + 600(15.71-30)^2 + \frac{80\times10^3}{12} + (15.71-5)^2$$

$$\boxed{I_{yy} = 0.4009\times10^6\ mm^4.}$$

$$I_{xy} = \Sigma Axy$$

$$= A_1 x_1 y_1 + A_2 x_2 y_2$$

$$= 600(50-30)(30.72-5) +$$

$$800(50-65)(30.72-50)$$

$x_1 = \bar{x} - x_1$

$y_1 = \bar{y} - y_1$

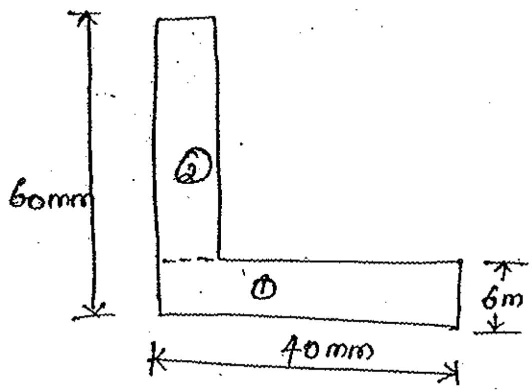
$x_2 = \bar{x} - x_2$

$y_2 = \bar{y} - y_2$

$$\boxed{I_{xy} = 0.53996\times10^6\ mm^4.}$$

Problem : 5

Calculate I_{xx}, I_{yy}, I_{xy} for angle Section $60\times40\times6$



$A_1 = 40\times6 = 240mm^2$

$A_2 = 54\times6 = 324\ mm^2$

$y_1 = \frac{6}{2} = 3mm$

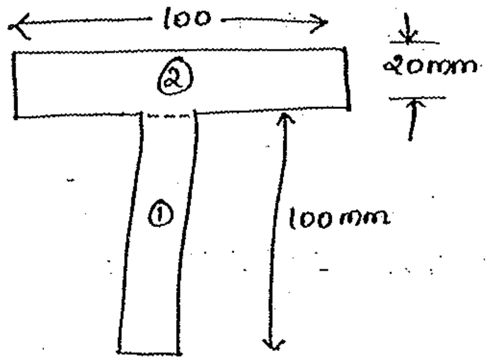
$y_2 = 6 + \frac{54}{2} = 33\ mm$

$x_1 = \frac{40}{2} = 20mm$

$x_2 = \frac{6}{2} = 3mm$

P-2.

A cast iron beam of "T" Section as shown in figure. A beam is Simply Supported on a Span of 8m. The beam carries a UDL of 1.5 KN/m throughout the length of entire beam. The maximum bending moment 12 MN-mm. Find the Bending Stress, where $t = 20$mm



Data give:-

$t = 20$ mm

$BM = 12$ MN-mm.

$A_1 = 100 \times 20 = 2000\,mm^2$ $\quad A_2 = 100 \times 20 = 2000\,mm^2$

$y_1 = \frac{100}{2} = 50\,mm$ $\quad y_2 = 100 + \frac{20}{2} = 110\,mm$.

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(2000 \times 50) + (2000 \times 110)}{2000 + 2000}$$

$\bar{y} = 80\,mm$

$$I_{xx} = I_1 + I_2 = \left[\frac{b_1 d_1^3}{12} + A_1 y_1^2\right] + \left[\frac{b_2 d_2^3}{12} + A_2 y_2^2\right]$$

$$= \left[\frac{20 \times 100^3}{12} + 2000(80-50)^2\right] + \left[\frac{100 \times 20^3}{12} + 2000(80-110)^2\right]$$

$$= 3.4666 \times 10^6 + 1.8666 \times 10^6$$

$I_{xx} = 5.3332 \times 10^6\,mm^4$.

Maximum bending stress at, $\bar{y} = y$

$\frac{M}{I} = \frac{\sigma}{y}$; $\sigma = 12 \times 10^6 \times 80 / 5.3332 \times 10^6$

$$\boxed{\sigma = 180.0045\ N/mm^2}$$

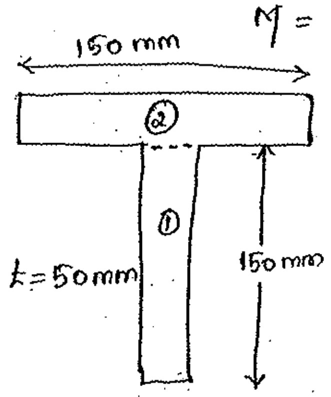
Problem - 3

A cast iron beam of T Section is Simply Supported on a Span of 4m. The beam Carries an UDL of 1.7 KN/m. Find the bending Stress, for the figure shown.

From given data.

Bending Moment for UDL $= \frac{Wl^2}{8} = \frac{1.7 \times 10^3 \times (4 \times 10^3)^2}{8}$

$M = 3.4 \times 10^9$ N-mm.



$A_1 = 150 \times 50 = 7500 \text{ mm}^2$

$A_2 = 150 \times 50 = 7500 \text{ mm}^2$

$y_1 = \frac{150}{2} = 75 \text{ mm}$

$y_2 = 150 + \frac{50}{2} = 170 \text{ mm}$.

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(7500 \times 75) + (7500 \times 170)}{7500 + 7500}$$

$\bar{y} = 125 \text{ mm}$.

$I_{xx} = I_1 + I_2$ (Since Symmetric about y axis)

$$= \frac{50 \times 150^3}{12} + 7500(125-75)^2 + \frac{150 \times 50^3}{12} + 7500(125-170)^2$$

ie $I_{xx} = \frac{b_1 d_1^3}{12} A_1(\bar{y} - y_1)^2 + \frac{b_2 d_2^3}{12} A_2(\bar{y} - y_2)^2$

$$\boxed{I_{xx} = 4.9562 \times 10^7 \text{ mm}^4.}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1+A_2} = \frac{240\times3 + 324\times33}{240 * 324}$$

$$\bar{y} = 20.23\ mm$$

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1+A_2} = \frac{240\times20 + 324\times3}{240+324}$$

$$\bar{x} = 10.234\ mm$$

$$I_{xx} = \frac{b_1d_1^3}{12} + A_1y_1^2 + \frac{b_2d_2^3}{12} + A_2y_2^2 \qquad y_1 = \bar{y} - y_1,\ x_2 = \bar{x} - x_2$$

$$= \frac{40\times6^3}{12} + 240\,(20.23-3)^2 + \frac{6\times54^3}{12} + 324\,(20.23-33)$$

$$\boxed{I_{xx} = 0.203537\times10^6\ mm^4.}$$

$$I_{yy} = \frac{b_1d_1^3}{12} + A_1x_1^2 + \frac{b_2d_2^3}{12} + A_2x_2^2$$

$$= \frac{60\times40^3}{12} + 240\,(10.234\times20)^2 + \frac{64\times6^3}{12} + 324\,(10.234-3)$$

$$\boxed{I_{yy} = 0.07281\times10^6\ mm^4}$$

$$I_{xy} = \Sigma Axy$$

$$= A_1x_1y_1 + A_2x_2y_2$$

$$= A_1(\bar{x}-x_1)(\bar{y}-y_1) + A_2(\bar{x}-x_2)(\bar{y}-y_2)$$

$$= 240(10.234-20)(20.23-3) + 324(10.234-3)(20.23-33)$$

$$\boxed{I_{xy} = 2.9543\times10^4\ mm^4.}$$

$$= \sigma \partial A . y$$

$$M = \int \sigma . y . dA$$

$$= \int \frac{E}{R} y^2 dA \qquad \because \sigma = \frac{E}{R} y$$

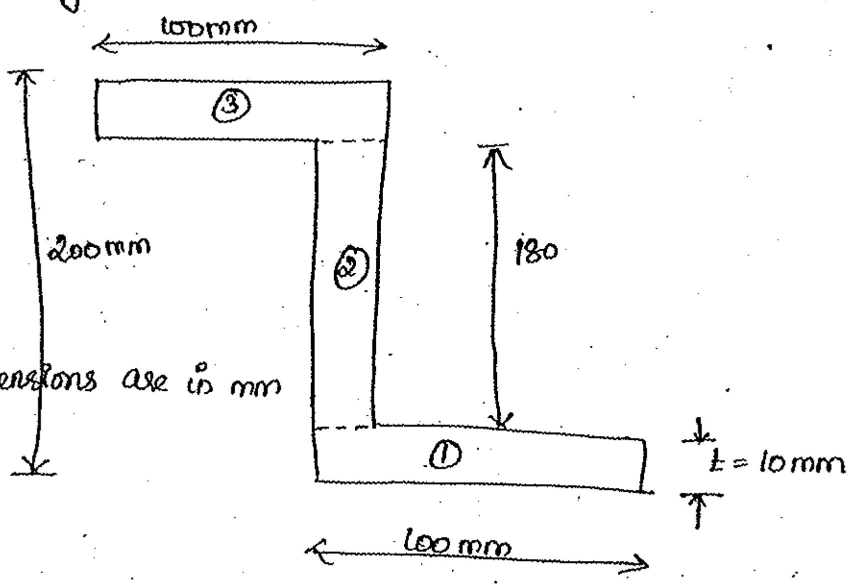
$$M = \frac{E}{R} I$$

$$\boxed{\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}}$$

This equation is said to be Bending equation.

Problem: 6

Find I_{xy} for the given section.



$A_1 = 100 \times 10 = 1000\,mm^2$ $\qquad A_2 = 180 \times 10 = 1800\,mm^2$ $\qquad A_3 = 100 \times 10 = 1000\,mm^2$

$y_1 = \frac{10}{2} = 5\,mm.$ $\qquad y_2 = 10 + \frac{180}{2} = 100\,mm$ $\qquad y_3 = 190 + \frac{10}{2} = 195\,mm$

$x_1 = 90 + \frac{100}{2} = 140\,mm$ $\qquad x_2 = 90 + \frac{10}{2} = 95\,mm$ $\qquad x_3 = \frac{100}{2} = 50\,mm$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2}$$

$$= \frac{(1000 \times 5) + (1800 \times 100) + (1000 \times 195)}{3800}.$$

$$\boxed{\bar{y} = 100 \text{ mm}}$$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3} = \frac{(1000 \times 140) + (1800 \times 95) + (1000 \times 50)}{3800}$$

$$\boxed{\bar{x} = 95 \text{ mm.}}$$

$$I_{xy} = \Sigma Axy = A_1 x_1 y_1 + A_2 x_2 y_2 + A_3 x_3 y_3$$

$$= 1000 \times (95 - 140) \times (100 - 5) + 1800 \times (95 - 95)(100 - 100)$$

$$+ 100 (95 - 50)(100 - 195)$$

$$= -4.275 \times 10^6 - 4.275 \times 10^6$$

$$\boxed{I_{xy} = -8.55 \times 10^6 \text{ mm}^4.}$$

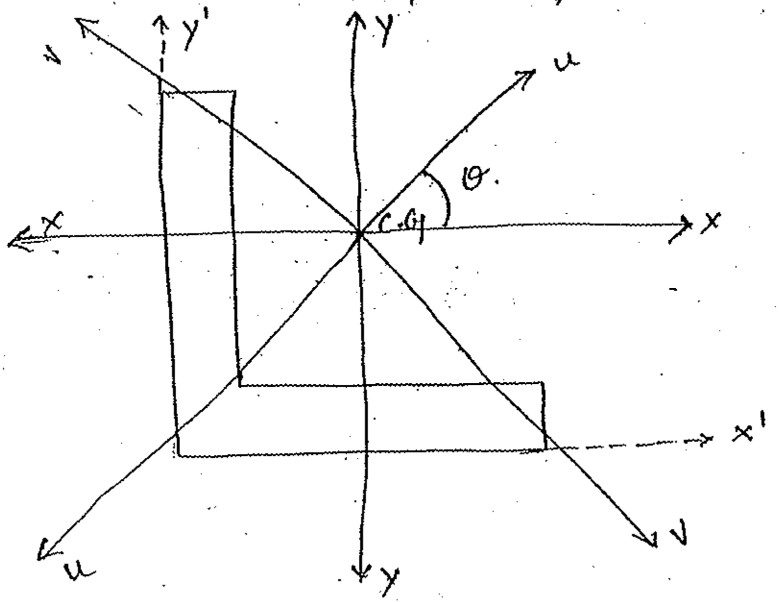
Different Methods to find Bending stress at a point in a Section

1. Principle axis method / principle moment of Inertia method
2. Neutral axis Method.
3. K - Method.

Method - 1

Principle moment of Inertia method :-

Moment of Inertia method is used to find the Bending Stresses of a whole beam where as principle moment of inertia method can be used to find the Bending Stress at certain points of the beam.



Here uu, vv are principal axis.

xx, yy are centroidal axis.

x', y' are reference axis.

where as $\sigma = M\left[\dfrac{v\cos\theta}{I_{uu}} + \dfrac{u\sin\theta}{I_{vv}}\right]$

$u = y\sin\theta + x\cos\theta$

$v = y\cos\theta - x\sin\theta$.

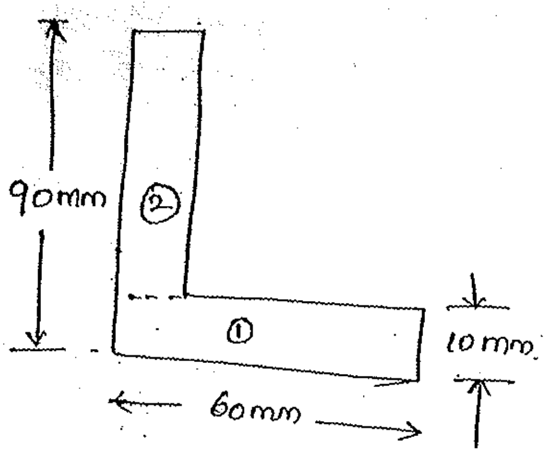
$\tan 2\theta = \dfrac{2I_{xy}}{I_{yy} - I_{xx}}$.

$$I_{uu} = \frac{I_{xx}+I_{yy}}{2} + \sqrt{\left(\frac{I_{yy}-I_{xx}}{2}\right)^2 + I_{xy}^2}$$

$$I_{vv} = \frac{I_{xx}+I_{yy}}{2} - \sqrt{\left(\frac{I_{yy}-I_{xx}}{2}\right)^2 + I_{xy}^2}$$

Problem - 8

Determine the principle moment of Inertia of a unequal angle section 90 x 60 x 10.



$A_1 = 60 \times 10 = 600 \text{ mm}^2$ $\quad A_2 = 80 \times 10 = 800 \text{ mm}^2$

$y_1 = \frac{10}{2} = 5 \text{ mm}.$ $\quad y_2 = 10 + \frac{80}{2} = 50 \text{ mm}$

$x_1 = \frac{60}{2} = 30 \text{ mm}$ $\quad x_2 = \frac{10}{2} = 5 \text{ mm}.$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(600 \times 5) + (800 \times 50)}{1400}$$

$$\bar{y} = 30.71 \text{ mm}$$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} = \frac{(600 \times 30) + (800 \times 5)}{1400}$$

$$\bar{x} = 15.71 \text{ mm}.$$

$$I_{xx} = I_{xx_1} + I_{xx_2}$$

$$= \frac{b_1 d_1^3}{12} + A_1 y_1'^2 + \frac{b_2 d_2^3}{12} + A_2 y_2'^2 \quad ; \quad y_1' = \bar{y} - y_1, \quad y_2' = \bar{y} - y_2$$

$$= \frac{60 \times 10^3}{12} + 600(30.71 - 5)^2 + \frac{10 \times 80^3}{12} + 800(30.71 - 50)^2$$

$$I_{xx} = 1.1259 \times 10^6 \text{ mm}^4$$

$$I_{yy} = I_{yy_1} + I_{yy_2}$$

$$= \frac{d_1 b_1^3}{12} + A_1 x_1'^2 + \frac{d_2 b_2^3}{12} + A_2 x_2'^2$$

$x_1' = \bar{x} - x_1$
$x_2' = \bar{x} - x_2$

$$= \frac{10 \times 60^3}{12} + 600\,(15.71 - 30)^2 + \frac{80 \times 10^3}{12} + 800\,(15.71 - 5)^2$$

$$I_{yy} = 0.401 \times 10^6 \text{ mm}^4.$$

$$I_{xy} = \Sigma Axy = A_1 x_1 y_1 + A_2 x_2 y_2$$

$x_1 = \bar{x} - x_1$
$x_2 = \bar{x} - x_2$
$y_1 = \bar{y} - y_1$
$y_2 = \bar{y} - y_2$

$$= 600(15.71 - 30)(30.71 - 5) + 800(15.71 - 5)(30.71 - 50)$$

$$I_{xy} = -0.3857 \times 10^6 \text{ mm}^4$$

$$I_{uu} = \frac{I_{xx} + I_{yy}}{2} + \sqrt{\left(\frac{I_{yy} - I_{xx}}{2}\right)^2 + I_{xy}^2}$$

$$= \frac{1.1259 \times 10^6 + 0.401 \times 10^6}{2} + \sqrt{\left(\left(\frac{0.401 - 1.1259}{2}\right) \times 10^6\right)^2 + \left(-0.3857 \times 10^6\right)^2}$$

$$I_{uu} = 1.2927 \times 10^6 \text{ mm}^4$$

$$I_{vv} = \frac{I_{xx} + I_{yy}}{2} - \sqrt{\left(\frac{I_{yy} - I_{xx}}{2}\right)^2 + I_{xy}^2}$$

$$= \frac{1.1259 \times 10^6 + 0.401 \times 10^6}{2} - \sqrt{\left(\left(\frac{0.401 - 1.1259}{2}\right) \times 10^6\right) + \left(-0.3857 \times 10^6\right)^2}$$

$$I_{vv} = 0.23417 \times 10^{6} \text{ mm}^4.$$

$$\tan 2\theta = \frac{2 I_{xy}}{I_{yy} - I_{xx}} = \frac{2 \times -0.3857 \times 10^{6}}{(0.401 - 1.1259) \times 10^{6}}$$

$$\tan 2\theta = 1.0641$$

$$2\theta = 46.779$$

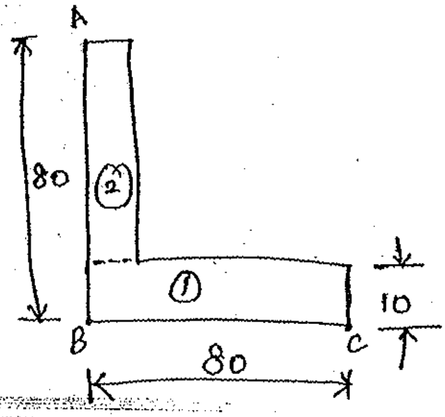
$$\theta = 23.339^{\circ}$$

To check

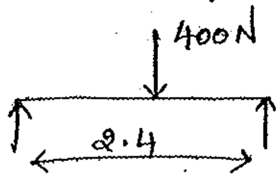
$$I_{uu} + I_{vv} = I_{xx} + I_{yy}.$$

Problem:- 8

A 80×80×10 mm angle section carries a load of 400 N and it is a Simple Supported Beam of span 2.4 m. Calculate Stresses at the point A, B & C.



For Bending Moment



$$M = \frac{W}{2} \times \frac{l}{2} = \frac{Wl}{4}$$

$$= \frac{400 \times 2.4}{4}$$

$$M = 240 \text{ N/m}.$$

$A_1 = 80 \times 10 = 800$ mm

$A_2 = 70 \times 10 = 700$ mm.

$\bar{y}_1 = \frac{10}{2} = 5mm$; $y_2 = 10 + \frac{70}{2} = 45mm$

$x_1 = \frac{80}{2} = 40mm$; $x_2 = \frac{10}{2} = 5mm$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(800 \times 5) + (700 \times 45)}{800 + 700}$$

$$\bar{y} = 23.67 \text{ mm}.$$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} = \frac{(800 \times 40) + (700 \times 5)}{800 + 700}$$

$$\bar{x} = 23.67 \text{ mm}$$

$$I_{xx} = I_{xx_1} + I_{xx_2}$$

$$= \frac{b_1 d_1^3}{12} + A_1 x_1'^2 + \frac{b_2 d_2^3}{12} + A_2 x_2'^2$$

$$= \frac{80 \times 10^3}{12} + 800 \times (23.67 - 5)^2 + \frac{10 \times 70^3}{12} + 700 (23.67 - 45)^2$$

$$I_{xx} = 0.8898 \times 10^6 \text{ mm}^4$$

$$I_{yy} = I_{yy_1} + I_{yy_2}$$

$$= \frac{d_1 b_1^3}{12} + A_2 y_1'^2 + \frac{d_2 b_2^3}{12} + A_3 y_2'^2$$

$$= \frac{10 \times 80^3}{12} + 800 (23.67 - 40)^2 + \frac{70 \times 10^3}{12} + 700 (23.67 - 5)^2$$

$I_{yy} = 0.8898 \times 10^6 \text{ mm}^4.$

$I_{xy} = \Sigma Axy = A_1x_1y_1 + A_2x_2y_2$

$= 800(23.67-40)(23.67-5) + 700(23.67-5)(23.67-45)$

$I_{xy} = 0.5226 \times 10^6 \text{mm}^4.$

$$I_{uu} = \frac{I_{xx}+I_{yy}}{2} + \sqrt{\left(\frac{I_{yy}-I_{xx}}{2}\right)^2 + I_{xy}^2}$$

$$= \frac{17.796 \times 10^5}{2} + \sqrt{0 + (-0.5226 \times 10^6)^2}$$

$I_{uu} = 1.4124 \times 10^6 \text{mm}^4$

$$I_{vv} = \frac{I_{xx}+I_{yy}}{2} - \sqrt{\left(\frac{I_{yy}-I_{xx}}{2}\right)^2 + I_{xy}^2}$$

$$= \frac{17.796 \times 10^5}{2} - \sqrt{0 + (-0.5226 \times 10^6)^2}$$

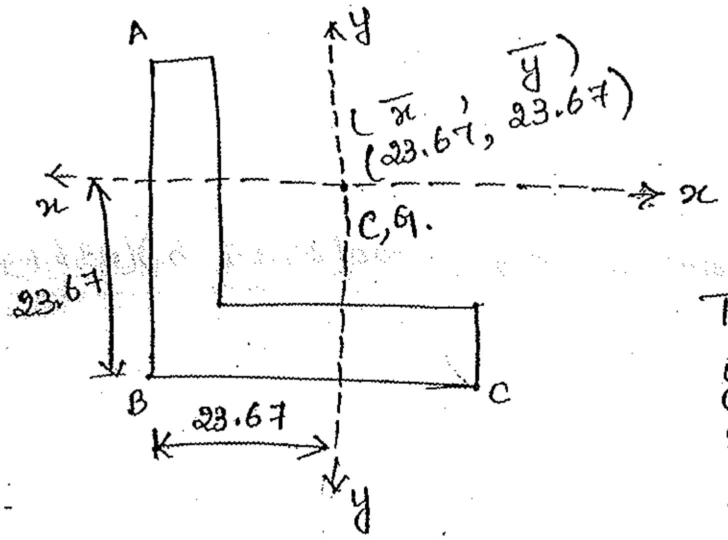
$I_{vv} = 0.3672 \times 10^6 \text{mm}^4$

$$\tan 2\theta = \frac{2I_{xy}}{I_{yy}-I_{xx}} = \frac{2I_{xy}}{0} = \infty$$

$\tan 2\theta = \infty$

$2\theta = 90°$

$\theta = 45°.$



To find u, V values you must measure the Point A, B, c from Centroidal axis.

Position from centroidal axis.

Point	(x)	(y)
A	-23.67mm	56.33 mm
B	-23.67mm	-23.67mm
C	56.33mm	-23.67mm

For A

$$\sigma_A = M\left[\frac{V\cos\theta}{I_{uu}} + \frac{u\sin\theta}{I_{vv}}\right] \; ; \; \begin{array}{l} V = y\cos\theta - x\sin\theta \\ u = y\sin\theta + x\cos\theta \end{array}$$

$$= 240 \times 10^3 \left[\frac{56.54 \cos 45^\circ}{1.412 \times 10^6} + \frac{23.09 \sin 45^\circ}{0.3672 \times 10^6}\right]$$

$$\sigma_A = 17.466 \ N/mm^2.$$

For B

$$\sigma_B = M\left[\frac{V\cos\theta}{I_{uu}} + \frac{u\sin\theta}{I_{vv}}\right]$$

$$V = (y\cos\theta - x\sin\theta) \; ; \; u = y\sin\theta + x\cos\theta.$$

$$V = 23.67 \cos 45^\circ + 23.67 \sin 45^\circ$$

$$V = 0.$$

$$u = -23.67 \sin 45^\circ - 23.67 \cos 45^\circ$$

$$u = -33.47 \text{ mm}$$

$$\sigma_B = 240 \times 10^3 \left[0 - \frac{33.47 \times \sin 45^\circ}{0.3672 \times 10^6} \right]$$

$$\sigma_B = -15.46 \text{ N/mm}^2$$

For c :-

$$u = y \sin\theta + x \cos\theta$$

$$= \cancel{56.33 \sin 45^\circ} \; -23.67 \sin 45 + 56.33 \cos 45^\circ$$

$$= 23.09 \text{ mm}.$$

$$V = y\cos\theta - x\sin\theta = -23.67 \cos 45^\circ - 56.33 \sin 45^\circ$$

$$V = -56.56 \text{ mm}.$$

$$\sigma_c = M \left[\frac{V\cos\theta}{I_{uu}} + \frac{u \sin\theta}{I_{vv}} \right]$$

$$= 240 \times 10^3 \left[\frac{-56.56 \cos 45}{1.2927 \times 10^6} + \frac{23.09 \times \sin 45^\circ}{0.23417 \times 10^6} \right]$$

$$\sigma_c = 9.3 \text{ N/mm}^2$$

Results

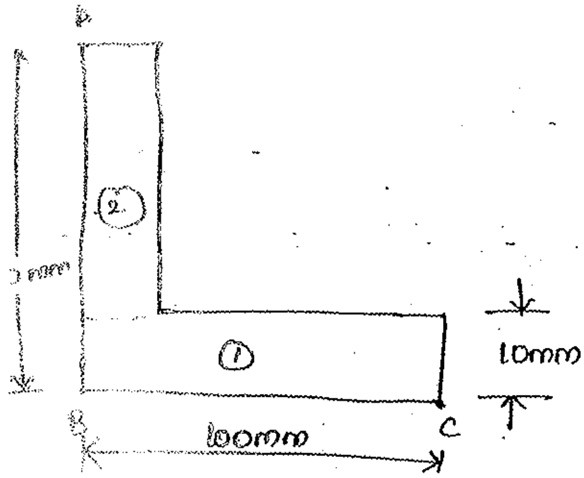
$\sigma_A = 17.46 \text{ N/mm}^2$

$\sigma_B = -15.46 \text{ N/mm}^2$

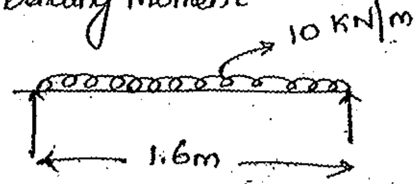
$\sigma_c = 9.3 \text{ N/mm}^2$

Problem:

A beam of angle section 150 x 100 x 10 is Simply Supported over Span of 1.6m with 150 mm leg Vertical. A UDL of 10 KN/m is pplied throughout the Span. Find the bending Stresses at location A, B & C.



Bending moment



$$M = \frac{wl^2}{8}$$

$$= \frac{10 \times 1.6^2}{8}$$

$$M = 3.2 \times 10^6 \text{ N-mm}$$

$A_1 = 100 \times 10 = 1000 mm^2$ $\qquad$ $A_2 = 140 \times 10 = 1400 mm^2$

$x_1 = \frac{100}{2} = 50mm$ $\qquad$ $x_2 = \frac{10}{2} = 5mm$

$y_1 = \frac{10}{2} = 5mm$ $\qquad$ $y_2 = 10 + \frac{140}{2} = 80mm$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(1000 \times 5) + (1400 \times 80)}{2400}$$

$$\bar{y} = 48.75 mm.$$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} = \frac{(1000 \times 50) + (1400 \times 5)}{2400}$$

$$\bar{x} = 23.75 mm.$$

$$I_{xx} = I_{xx_1} + I_{xx_2}$$

$$= \frac{b_1 d_1^3}{12} + A_1 y_1'^2 + \frac{b_2 d_2^3}{12} + A_2 y_2'^2$$

$y_1' = \bar{y} - y_1$
$y_2' = \bar{y} - y_2$
$x_1' = \bar{x} - x_1$
$x_2' = \bar{x} - x_2$

$$= \frac{100 \times 10^3}{12} + 1000(48.75-5)^2 + \frac{10 \times 140^3}{12} + 1400(48.75-80)^2$$

$$I_{xx} = 5.576 \times 10^6 \text{ mm}^4.$$

$$I_{yy} = \frac{d_1 b_1^3}{12} + A_1 x_1'^2 + \frac{d_2 b_2^3}{12} + A_2 x_2'^2$$

$$= \frac{10 \times 100^3}{12} + 1000\,(23.75-50)^2 + \frac{140 \times 10^3}{12} + 1400\,(23.75-5)^2$$

$$I_{yy} = 2.026 \times 10^6 \text{ mm}^4$$

$$I_{xy} = \Sigma Axy = A_1 x_1' y_1' + A_2 x_2' y_2'$$

$x_1' = \bar{x} - x_1$
$x_2' = \bar{x} - x_2.$

$$= 1000\,(23.75-50)\,(48.75-5) + 1400\,(23.75-5)(48.75-80)$$

$$I_{xy} = -1.968 \times 10^6 \text{ mm}^4.$$

$$\tan 2\theta = \frac{2 I_{xy}}{I_{yy} - I_{xx}} = \frac{2 \times 1.968 \times 10^6}{(2.026-5.57) \times 10^6}$$

$$\theta = 23.99^\circ.$$

$$I_{uu} = \frac{I_{xx} + I_{yy}}{2} + \sqrt{\left(\frac{I_{yy} - I_{xx}}{2}\right)^2 + I_{xy}^2}$$

$$= \frac{(5.57 + 2.026) \times 10^6}{2} + \sqrt{\left(\frac{2.026 - 5.57}{2}\right)^2 + (1.968 \times 10^6)^2}$$

JB

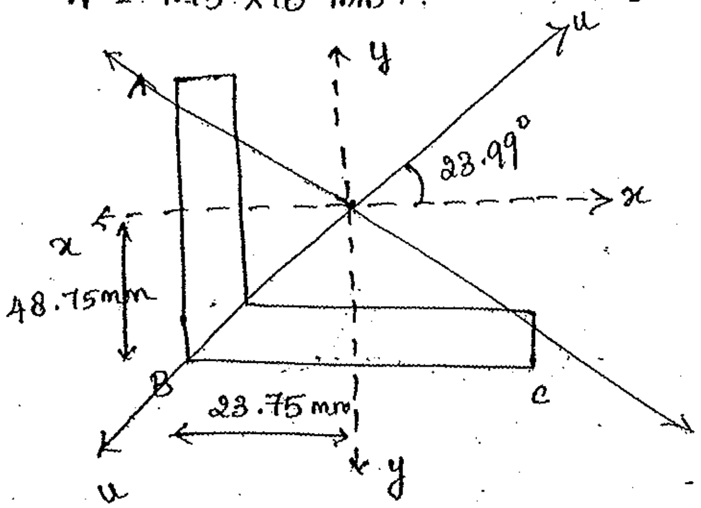
$$I_{uu} = 3.798 \times 10^6 \, mm^4.$$

$$I_{VV} = \frac{I_{xx} + I_{yy}}{2} - \sqrt{\left(\frac{I_{yy} - I_{xx}}{2}\right)^2 + I_{xy}^2}$$

$$= \frac{(5.57 + 2.026) \times 10^6}{2} - \sqrt{\left(\frac{2.026 - 5.57}{2}\right)^2 + \left(1.968 \times 10^6\right)^2}$$

$$= 3.798 \times 10^6 - 2.648 \times 10^6$$

$$I_{VV} = 1.15 \times 10^6 \, mm^4.$$



For A:

$x = -23.75 mm$; $y = 150 - 48.75 = 101.25 mm$

$$u = y \sin\theta + x \cos\theta$$

$$= 101.25 \sin 23.99 - 23.75 \cos 23.99$$

$$u = 19.467 \, mm$$

$$V = y \cos\theta - x \sin\theta$$

$$= 101.25 \cos 23.99 + 23.75 \sin 23.99$$

$$V = 102.159 \, mm$$

Jls.

Normal bending stress at A,

$$\sigma_A = M\left[\frac{V\cos\theta}{I_{uu}} + \frac{u\sin\theta}{I_{vv}}\right]$$

$$= 3.2\times10^6\left[\frac{102.159\times\cos 23.99}{6.446\times10^6} + \frac{19.467\times\sin 23.99}{1.159\times10^6}\right]$$

$$\sigma_A = 68.35\ N/mm^2$$

For B :

$$x = -23.75\,mm\ ;\ y = -48.75\,mm$$

$$u = y\sin\theta + x\cos\theta = -48.75\sin 23.99 - 23.75\cos 23.99$$

$$u = -41.519$$

$$V = y\cos\theta - x\sin\theta = -48.75\cos 23.99 + 23.75\sin 23.99$$

$$V = -34.88$$

Normal bending stress at B,

$$\sigma_B = M\left[\frac{V\cos\theta}{I_{uu}} + \frac{u\sin\theta}{I_{vv}}\right]$$

$$= 3.2\times10^6\left[\frac{-34.88\times\cos 23.99}{6.446\times10^6} + \frac{-41.519\sin 23.99}{1.15\times10^6}\right]$$

$$= -3.2\left[4.943 + 14.678\right]$$

$$\sigma_B = -62.78\ N/mm^2.$$

For c :-

$$x = 100 - 23.75 = 76.25\,mm$$

$$y = -48.75\,mm$$

$u = y\sin\theta + x\cos\theta = -48.75 \sin 23.99 + 76.25 \cos 23.99$

$u = 49.84$

$v = y\cos\theta - x\sin\theta$

$= -48.75 \cos 23.99 - 76.25 \sin 23.99$

$= -44.538 - 31.00$

$v = -75.538$

Normal bending stress at c,

$$\sigma_c = M\left[\frac{v\cos\theta}{I_{uu}} + \frac{u\sin\theta}{I_{vv}}\right]$$

$$= 3.2\times10^6\left[\frac{-75.538 \cos 3.99}{6.446\times10^6} + \frac{49.84\times\sin 23.99}{1.15\times10^6}\right]$$

$$\sigma_c = 22.127\ N/mm^2$$

Method : 2

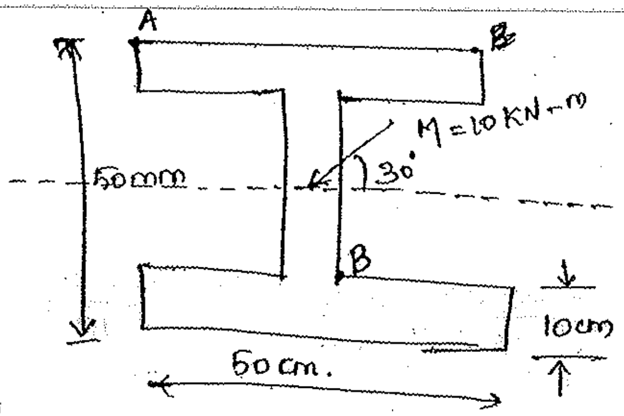
"k" Method.

Normal bending stress at any point is

$$\sigma_z = \frac{\left[(M_y I_{xx} - M_x I_{xy})x + (M_x I_{yy} - M_y I_{xy})y\right]}{I_{xx} I_{yy} - I_{xy}^2}$$

Problem 10 :-

Find the bending stress at location A & B of I Section and calculate the angle, a 10 KN-m Bending moment acts at an angle of 30° as shown in figure.



solution

$$M = 10\ KN\text{-}m.$$

$$M_x = M\cos\theta = 10 \times \cos 30^\circ$$

$$M_x = 8.66\ KN\text{-}m = 8.66 \times 10^5\ N\text{-}cm.$$

$$M_y = M\sin\theta = 10 \times \sin 30^\circ$$

$$M_y = 5\ KN\text{-}m = 5 \times 10^5\ N\text{-}cm$$

$A_1 = 50 \times 10 = 500\ cm^2$ $\quad A_2 = 30 \times 10 = 300\ cm^2$ $\quad A_3 = 50 \times 10 = 500\ cm^2$

$x_1 = \frac{50}{2} = 25\ cm$ $\quad x_2 = \frac{10}{2} + 20 = 25\ cm$ $\quad x_3 = \frac{50}{2} = 25\ cm$

$y_1 = \frac{10}{2} = 5\ cm$ $\quad y_2 = 10 + \frac{30}{2} = 25\ cm$ $\quad y_3 = 40 + \frac{10}{2} = 45\ cm$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{(500)(5) + (300 \times 25) + (500 \times 45)}{500 + 300 + 500}$$

$$\bar{y} = 25\ cm$$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

$$= \frac{(500 \times 25) + (300 \times 25) + (500 \times 25)}{500 + 300 + 500}$$

$$\bar{x} = 25\ cm.$$

$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$$

$$= \frac{b_1 d_1^3}{12} + A y_1'^2 + \frac{b_2 d_2^3}{12} + A_2 y_2'^2 + \frac{b_3 d_3^3}{12} + A_3 y_3'^2$$

$$= \frac{50 \times 10^3}{12} + 500(25-5)^2 + \frac{10 \times 30^3}{12} + 300(25-25)^2 + \frac{50 \times 10^3}{12} + 500(25-45)^2$$

$$I_{xx} = 4.3 \times 10^5 \text{ cm}^4.$$

$$I_{yy} = I_{yy_1} + I_{yy_2} + I_{yy_3}$$

$$= \frac{d_1 b_1^3}{12} + A_1 x_1'^2 + \frac{d_2 b_2^3}{12} + A_2 x_2'^2 + \frac{d_3 b_3^2}{12} + A_3 x_3'^2$$

$$= \frac{10 \times 50^3}{12} + 500(25-25)^2 + \frac{30 \times 10^3}{12} + 300(25-25) + \frac{10 \times 50^3}{12} + 500(25-25)^2$$

$$I_{yy} = 2.1 \times 10^5 \text{ cm}^4$$

$$I_{xy} = \Sigma A x y = A_1 x_1' y_1' + A_2 x_2' y_2' + A_3 x_3' y_3'$$

$$= 500(25-25)(25-5) + 300(25-25)(25-25) + 500(25-25)(25-45)$$

$x_1' = \bar{x} - x_1$
$x_2' = \bar{x} - x_2$
$x_3' = \bar{x} - x_3$
$y_1' = \bar{y} - y_1$
$y_2' = \bar{y} - y_2$
$y_3' = \bar{y} - y_3$

$$I_{xy} = 0.$$

To find Bending stress:-

$$\sigma_x = \left[\frac{(M_y I_{xx} - M_x I_{xy})x + (M_x I_{yy} - M_y I_{xy})y}{I_{xx} I_{yy} - I_{xy}^2} \right]$$

For A :- $x = -25$ cm $y = 25$ cm

18.186×10^{10}

454×10^{10} $- 8.66 \times 10$

$$\sigma_A = \left[\frac{(5 \times 10^5 \times 4.3 \times 10^5 - 8.66 \times 10^5 \times 0)25 + (8.66 \times 10^5 \times 2.1 \times 10^5 - 0)25}{4.3 \times 10^5 \times 2.1 \times 10^5 - 0} \right]$$

$$\frac{-537.5 \times 10^{10} + 454.65 \times 10^{10}}{9.45 \times 10^{10}}$$

$$= \frac{(\cancel{58.25}^{21.5} \times 10^{10})x + (18.18 \times 10^{10})y}{9.03 \times 10^{10}}$$

$$\sigma_A = \cancel{909.85}^{-0.367} \; N/cm^2$$

For B:

$x = 5cm$; $y = -15 cm$

$$\sigma_B = -\left[\frac{(5 \times 4.3 \times 10^5 \times 10^5 - 0)5 + (8.66 \times 10^5 \times 2.1 \times 10^5 - 0)15}{4.3 \times 10^5 \times 2.1 \times 10^5 - 0}\right]$$

$\sigma_B = 42.10 \; N/cm^2$ 42.10

To find β:

To find β, equate any one of normal bending stress to zero

$$\frac{(5 \times 4.3 \times 10^5 \times 10^5)x + (8.66 \times 10^5 \times 2.1 \times 10^5)y}{4.3 \times 10^5 \times 2.1 \times 10^5} = 0$$

$$21.5x + 18.186y = 0$$

$$21.5x = -18.186y$$

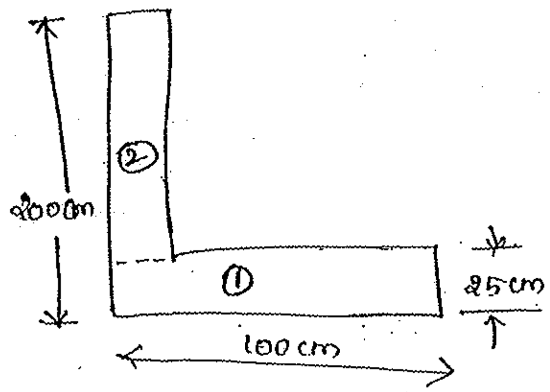
$$\frac{y}{x} = -1.182$$

$$\beta = \tan^{-1}(y/x) = \tan^{-1}(-1.182)$$

$$\beta = -49.76^\circ$$

Problem: 1

A given angle Section 200×100×25cm is Subjected to a moment $M_x = 10$ KN-m. Find the bending Stresses at Corner points.



$M_x = 10$ KN-m
$= 10 \times 10^5$ N-cm

$M_y = 0$.

$A_1 = 100 \times 25 = 2500\ cm^2$ $\qquad$ $A_2 = 175 \times 25 = 4375\ cm^2$

$y_1 = \frac{25}{2} = 12.5$ cm $\qquad$ $y_2 = 25 + \frac{175}{2} = 112.5$ cm

$x_1 = \frac{100}{2} = 50$ cm. $\qquad$ $x_2 = \frac{25}{2} = 12.5$ cm.

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(2500)(12.5) + (4375)(112.5)}{2500 + 4375}$$

$\bar{y} = 76.13$ cm.

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} = \frac{(2500)(50) + (4375)(12.5)}{2500 + 4375}$$

$\bar{x} = 26.13$ cm.

$$I_{xx} = I_{xx_1} + I_{xx_2}$$

$$= \frac{b_1 d_1^3}{12} + A_1 y_1'^2 + \frac{b_2 d_2^3}{12} + A_2 y_2'^2$$

$y_1' = \bar{y} - y_1$
$y_2' = \bar{y} - y_2$

$$= \frac{100 \times 25^3}{12} + 2500(76.13 - 12.5)^2 + \frac{25 \times 175^3}{12} + 4375(76.13 - 112.5)^2$$

$I_{xx} = 27.2 \times 10^6 \text{ cm}^4$

$$I_{yy} = \frac{d_1 b_1^3}{12} + A_1 x_1'^2 + \frac{d_2 b_2^3}{12} + A_2 x_2'^2$$

$$= \frac{25 \times 100^3}{12} + 2500\,(26.13-50)^2 + \frac{175 \times 25^3}{12} + 4375\,(26.13-125)^2$$

$I_{yy} = 4.548 \times 10^6 \text{ cm}^4.$

$$I_{xy} = \Sigma Axy = A_1 x_1' y_1' + A_2 x_2' y_2'$$

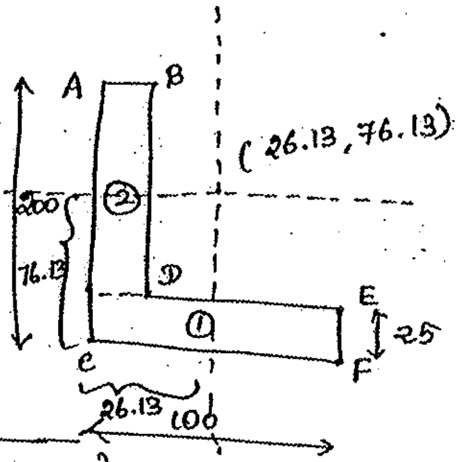
$$= A_1(\bar{x}-x_1)(\bar{y}-y_1) + A_2(\bar{x}-x_2)(\bar{y}-y_2)$$

$$= 2500\,(26.13-50)(76.13-12.5) + 4375\,(26.13-12.5)(76.13-112.5)$$

$I_{xy} = -5.965 \times 10^6 \text{ cm}^4.$

$$\sigma = \left[\frac{(M_y I_{xx} - M_x I_{xy})x + (M_x \cdot I_{yy} - M_y \cdot I_{xy})y}{I_{xx} \cdot I_{yy} - I_{xy}^2}\right]$$

<u>At A</u> :



$x = -26.13$; $y = 123.87$

$$\sigma_A = \frac{\left[-\left(0 + 10 \times 10^5 \times 5.965 \times 10^6\right) \times 26.13\right] + \left[\left(10 \times 10^5 \times 4.548 \times 10^6 - 0\right) \times 123.87\right]}{27.2 \times 10^6 \times 4.548 \times 10^6 - (5.965 \times 10^6)^2}$$

$\sigma_A = 4.624 \text{ N/cm}^2$

At B :-

$$x = -1.13 \; ; \; y = +76.13$$

$$\sigma_B = \left[\frac{(5.966 \times 10^{1})(-1.136) + (4.548 \times 10^{12})(123.864)}{88.1124 \times 10^{12}} \right]$$

$$\sigma_B = 6.32 \times 10^{-3} \, KN/cm^2 = 6.32 \, N/cm^2$$

At C :-

$$x = -26.136 \; ; \; y = -76.13$$

$$\sigma_C = \left[\frac{(5.966 \times 10^{12})(-26.136) + (4.548 \times 10^{12})(-76.136)}{88.1124 \times 10^{12}} \right]$$

$$\sigma_C = -5.649 \times 10^{-3} \, KN/cm^2 = -5.649 \, N/cm^2$$

At D :- $x = -1.136 \; ; \; y = -51.136$

$$\sigma_D = \left[\frac{(5.966 \times 10^{12})(-1.136) + (4.548 \times 10^{12})(-51.136)}{88.1124 \times 10^{12}} \right]$$

$$\sigma_D = -2.766 \times 10^{-3} \, KN/cm^2 = -2.766 \, N/cm^2$$

t E :- $x = 73.64 \; ; \; y = -51.136$

$$\sigma_E = \left[\frac{(5.966 \times 10^{12})(73.64) + (4.548 \times 10^{12})(-51.136)}{88.1124 \times 10^{12}} \right]$$

$$\sigma_E = 2.347 \times 10^{-3} \, KN/cm^2 = 2.347 \, N/cm^2$$

t F :- $x = 73.64 \; ; \; y = -76.136$

$$\sigma_F = \left[\frac{(5.966 \times 10^{12})(73.64) + (4.548 \times 10^{12})(-76.136)}{88.1124 \times 10^{12}} \right]$$

$\sigma_F = -1.056 \times 10^{-3}\, KN/cm^2$; $\sigma_F = -1.056\ N/cm^2$

To find β :-

Consider $\sigma_A = 0$.

$$(+5.966 \times 10^{6})x + (4.548 \times 10^{6})y = 0$$

$$\frac{y}{x} = -\frac{5.966}{4.548}$$

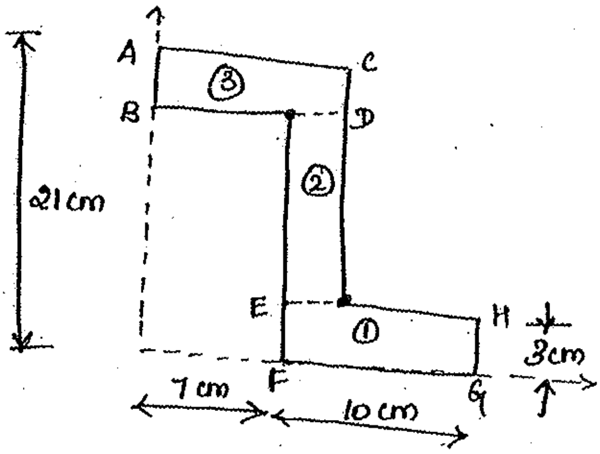
$$\tan\beta = -1.312$$

$$\beta = \tan^{-1}(-1.312)$$

$$\beta = -52.69^\circ.$$

Problem : 12

Find the Bending stress at the corner points of Z-section with flanges 10 x 3 cm & web 15 x 3 cm Subjected to moments $M_x = 10\,KNm$ & $M_y = 5\,KNm$. Also calculate the neutral axis.



$M_x = 10\,KN\text{-}m = 10 \times 10^3 \times 10^2\, Ncm$

$= 10 \times 10^5\, N\text{-}cm$

$M_y = 5\,KN\text{-}m$

$= 5 \times 10^5\, N\text{-}cm.$

$A_1 = 10 \times 3 = 30\,cm^2$ $\quad A_2 = 15 \times 3 = 45\,cm^2$ $\quad A_3 = 10 \times 3 = 30\,cm^2$

$y_1 = \frac{3}{2} = 1.5\,cm$ $\quad y_2 = 3 + \frac{15}{2} = 10.5\,cm$ $\quad y_3 = 18 + \frac{3}{2} = 19.5\,cm$

$x_1 = 7 + \frac{10}{2} = 12\,cm$ $\quad x_2 = 7 + \frac{3}{2} = 8.5\,cm$ $\quad x_3 = \frac{10}{2} = 5\,cm$

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1+A_2+A_3} = \frac{(30\times12)+(45\times8.5)+(30\times5)}{30+45+30}$$

$$\bar{x} = 8.5\,cm$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1+A_2+A_3} = \frac{(30\times1.5)(45\times10.5)+(30\times19.5)}{30+45+30}$$

$$\bar{y} = 10.5\,cm$$

$$I_{xx} = \frac{b_1d_1^3}{12} + A_1{y_1'}^2 + \frac{b_2d_2^3}{12} + A_2{y_2'}^2 + \frac{b_3d_3^3}{12} + A_3{y_3'}^2$$

$$= \frac{10\times3^3}{12} + 30(10.5-1.5)^2 + \frac{3\times15^3}{12} + 45(10.5-10.5)^2 + \frac{10\times3^3}{12} + 30(10.5-19.5)^2$$

$$I_{xx} = 5.748\times10^3\,cm^4$$

$$I_{yy} = \frac{d_1b_1^3}{12} + A_1{x_1'}^2 + \frac{d_2b_2^3}{12} + A_2{x_2'}^2 + \frac{d_3b_3^3}{12} + A_3{x_3'}^2$$

$$= \frac{3\times10^3}{12} + 30(8.5-12)^2 + \frac{15\times3^3}{12} + 45(8.5-8.5)^2 + \frac{3\times10^3}{12} + 30(8.5-5)^2$$

$$I_{yy} = 1.26\times10^3\,cm^4$$

$$I_{xy} = \Sigma Axy = A_1x_1'y_1' + A_2x_2'y_2' + A_3x_3'y_3'$$

$$= 30(8.5-12)(10.5-1.5) + 45(8.5-8.5)(10.5-10.5) + 30(8.5-5)(10.5-19.5)$$

$$I_{xy} = -1.89\times10^3\,cm^4$$

At A: $x = -8.5, y = 10.5$

$$\sigma_A = \frac{\left[(M_y I_{xx} - M_x I_{xy})x + (M_x I_{yy} - M_y I_{xy})y\right]}{I_{xx} I_{yy} - I_{xy}^2}$$

$$= \frac{\left[(5\times10^5 \times 5748.75) - (10\times10^5 \times (-)1890)(-8.5) + (10\times10^5 \times 1268.75) - (5\times10^5 (-)(1890)(10.5))\right]}{5748.75 \times 1268.75 - (-1.89\times10^3)^2}$$

$$\sigma_A = 4.635 \times 10^3 \ N/cm^2$$

At B:- $x = -8.5$; $y = 7.5$

$$\sigma_B = \frac{\left[(5\times10^5 \times 5748.75) - (10\times10^5 \times (-)1890)(-8.5) + (10\times10^5 \times 1268.75) - (5\times10^5 (-)(1890)(7.5))\right]}{3.722\times10^6}$$

$$\sigma_B = -6.419\times10^3 \ N/cm^2$$

At C:- $x = 1.5$; $y = 10.5$

$$\sigma_c = \frac{\left[(5\times10^5 \times 5748.75) - (10\times10^5 \times (-)(1890)(-8.5) + (10\times10^5 \times 1268.75) - (5\times10^5 (-)(1890)(10.5))\right]}{3.722\times10^6}$$

$$\sigma_c = 8.176\times10^3 \ N/cm^2.$$

At D $\quad x = 1.5 \; ; \; y = 7.5$

$$\sigma_D = \frac{\left[(5\times10^5 \times 5748.75 + 10\times10^5 \times 1.89\times10^3)(1.5) + (10\times10^5 \times 1268.75 + 5\times10^5 \times 1.89\times10^3)(7.5) \right]}{3.722\times10^6}$$

$$\sigma_D = 6.383\times10^3 \; N/cm^2$$

At E $\quad x = -1.5 \; ; \; y = -7.5$

$$\sigma_E = \frac{\left[(5\times10^5 \times 5748.75 + 10\times10^5 \times 1.89\times10^3)(-1.5) + (10\times10^5 \times 1268.75 + 5\times10^5 \times 1.89\times10^3)(-7.5) \right]}{3.722\times10^6}$$

$$\sigma_E = 6.383\times10^3 \; N/cm^2$$

At F $\quad x = 1.5 \; ; \; y = -10.5$

$$\sigma_F = \frac{\left[(5\times10^5 \times 5748.75 + 10\times10^5 \times 1.89\times10^3)(1.5) + (10\times10^5 + 1268.75 + 5\times10^5 \times 1.89\times10^3)(-10.5) \right]}{3.722\times10^6}$$

$$\sigma_F = -8.164\times10^3 \; N/cm^2.$$

At G: $\quad x = 8.5 \; ; \; y = -10.5$

$$\sigma_G = \frac{\left[(5\times10^5 \times 5748.75 + 10\times10^5 \times 1.89\times10^3)(8.5) + (10\times10^5 + 1268.75 + 5\times10^5 \times 1.89\times10^3)(-10.5) \right]}{3.722\times10^6}$$

At H :- $x = 8.5$; $y = -7.5$

$$\sigma_H = -\frac{\left[(5\times10^5 \times 5748.75 + 10\times10^5 \times 1.89\times10^3)(8.5) + (10\times10^5 + 12.68.75 + 5\times10^5 \times 1.89\times10^3)(-7.5)\right]}{3.722\times10^6}$$

$$\sigma_H = 6.419\times10^3\ N/cm^2$$

To find β :-

Equate σ_A to 0.

$$(4.764\times10^9 x + 2.214\times10^9 y) = 0.$$

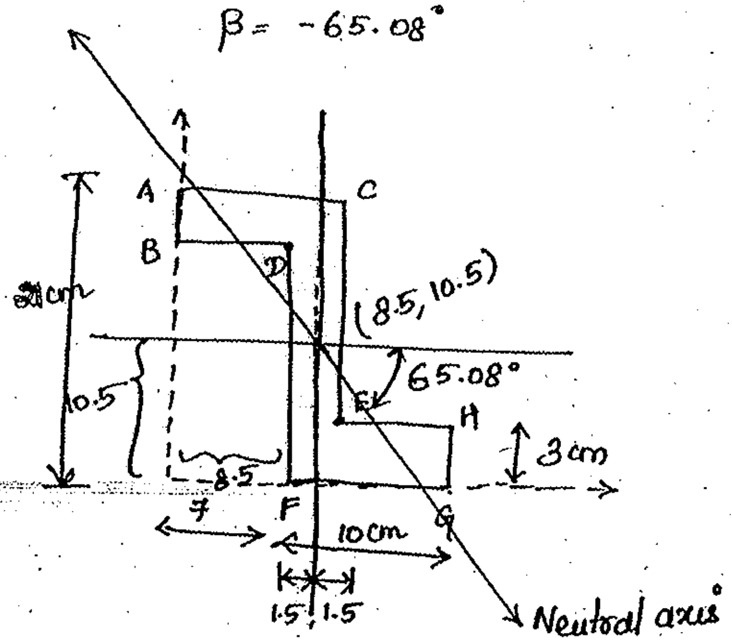
$$4.764\times10^9 x = -2.214\times10^9 y$$

$$y/x = -2.512$$

$$\beta = \tan^{-1}(y/x)$$

$$= -6.508\times10^1$$

$$\beta = -65.08^\circ$$



Values above Neutral axis will be negative & below Neutral axis will be positive.

Method :- 3

Neutral Axis Method

Bending stress at any point is

$$\sigma_z = \frac{M_n}{I_n} y_n$$

$$M_n = M_x \cos\alpha + M_y \sin\alpha$$

$$I_n = I_{xx}\cos^2\alpha + I_{yy}\sin^2\alpha - I_{xy}\sin 2\alpha$$

$$y_n = y\cos\alpha - x\sin\alpha$$

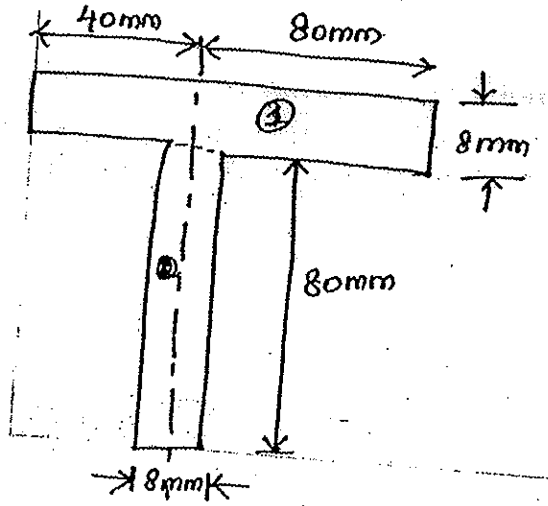
$$\tan\alpha = \left[-\frac{\bar{M}_y}{\bar{M}_x} \times \frac{I_{xx}}{I_{yy}}\right]$$

$$\bar{M}_y = \frac{M_y - M_x\left(\frac{I_{xy}}{I_{xx}}\right)}{1 - \frac{I_{xy}^2}{I_{xx} I_{yy}}}$$

$$\bar{M}_x = \frac{M_x - M_y\left(\frac{I_{xy}}{I_{yy}}\right)}{1 - \frac{I_{xy}^2}{I_{xx} I_{yy}}}$$

Problem :- 13

Find the Maximum normal stress due to bending for the section Shown in figure. $M_x = 1500\ N\text{-}m$; $M_y = 0$.



$A_1 = 120 \times 8 = 960\ mm^2$. $A_2 = 80 \times 80 = 640\ mm^2$.

$x_1 = \dfrac{120}{2} = 60\ mm$ $\qquad x_2 = 36 + \dfrac{8}{2} = 40\ mm$

$y_1 = 80 + \dfrac{8}{2} = 84\ mm$ $\qquad y_2 = \dfrac{80}{2} = 40\ mm$.

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} = \frac{\overset{57600}{(960 \times 60)} + \overset{25600}{(640 \times 40)}}{960 + 640 \ \ 1600}.$$

$\bar{x} = 52\ mm$.

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{\overset{80640}{(960 \times 84)} + \overset{25600}{(640 \times 40)}}{960 + 640}.$$

$\bar{y} = 66.4\ mm$

$$I_{xx} = I_{xx_1} + I_{xx_2}$$

$$= \frac{b_1 d_1^3}{12} + A_1(\bar{y} - y_1)^2 + \frac{b_2 d_2^3}{12} + A_2(\bar{y} - y_2)^2$$

297362.6   445054.4   696.96

$$= \frac{120 \times 8^3}{12} + 960(66.4-84)^2 + \frac{8\times 80^3}{12} + 640(66.4-40)^2$$

302.48×10^3 787.387×10^3

$$I_{XX} = 302.48\times 10^3 + 787.387 \times 10^3$$

1089.867×10^3

$$I_{xx} = 1.089 \times 10^6\ mm^4.$$

$$I_{yy} = \frac{d_1 b_1^3}{12} + A_1(\bar{x}-x_1)^2 + \frac{d_2 b_2^3}{12} + A_2(\bar{x}-x_2)^2$$

61440

$$= \frac{8\times 120^3}{12} + 960(52-60)^2 + \frac{80\times 8^3}{12} + 640(52-40)^2$$

92160

1.213×10^6 0.0957×10^6

$$= 1.213\times 10^6 + 0.0957 \times 10^6$$

$$I_{yy} = 1.309 \times 10^6\ mm^4.$$

$$I_{xy} = \Sigma AXY$$

$$= A_1(\bar{x}-x_1)(\bar{y}-y_1) + A_2(\bar{x}-x_2)(\bar{y}-y_2)$$

$$= 960(52-60)(66.4-84) + 640(52-40)(66.4-40)$$

$$= 3.379 \times 10^5\ mm^4$$

$$I_{xy} = 0.3379 \times 10^6\ mm^4.$$

$$\overline{M}_x = \frac{M_x - M_y\left(\frac{I_{xy}}{I_{yy}}\right)}{1 - \frac{I_{xy}^2}{I_{xx} I_{yy}}}$$

$\because M_x = 150 \times 10^3\ N\text{-}mm$

$M_y = 0$

$$= \frac{150\times 10^3 - 0}{1 - \frac{(3.379\times 10^5)^2}{1.089\times 10^6 \times 1.309 \times 10^6}} = 1.631 \times 10^6\ N\text{-}mm$$

$$\bar{M_y} = \frac{M_y - M_x\left(\frac{I_{xy}}{I_{xx}}\right)}{1 - \frac{I_{xy}^2}{I_{xx} I_{yy}}} = \frac{0 - 1500\times10^3\left(\frac{3.379\times10^5}{1.089\times10^6}\right)}{1 - \frac{(3.379\times10^5)^2}{1.089\times10^6\times1.309\times10^6}}$$

0.0800956

$$\bar{M_y} = -5.059\times10^6 \text{ N-mm.}$$

$$\tan\alpha = \left[-\frac{\bar{M_y}}{\bar{M_x}} \cdot \frac{I_{xx}}{I_{yy}}\right]$$

$$= \left[\frac{5.059\times10^6}{1.631\times10^6} \times \frac{1.089\times10^6}{1.309\times10^6}\right]$$

$$\alpha = 14.47^\circ.$$

$$M_n = M_x \cos\alpha + M_y \sin\alpha$$

$$= 1500\times10^3 \times \cos 14.47^\circ$$

$$= 1.4524\times10^6 \text{ N-mm.}$$

$$I_n = I_{xx}\cos^2\alpha + I_{yy}\sin^2\alpha - I_{xy}\sin 2\alpha$$

$$I_n = 1.089\times10^6\cos^2 14.47 + 1.309\times10^6\times\sin^2 14.47$$
$$- 3.379\times10^5 \sin 2(14.47)$$

$$I_n = 9.40\times10^5 \text{ mm}^4.$$

$$\sigma_z = \frac{M_n}{I_n} y_n$$

At Points.

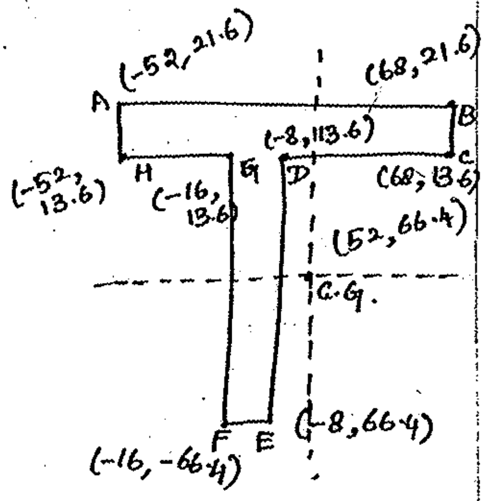
$$\sigma_x = \frac{1.4524 \times 10^6}{9.400 \times 10^5} \, y_n$$

$$= 1.54 \, y_n$$

$$= 1.54 \left[y \cos\alpha - x \sin\alpha \right]$$

Location of points with respect to centroid

Point	x	y
A	-52	21.6
B	68	21.6
C	68	13.6
D	-8	13.6
E	-8	-66.4
F	-16	-66.4
G	-16	13.6
H	-52	13.6



$$(\sigma_z)_A = 1.54 \left[21.6 \cos 14.47 + 52 \sin 14.47 \right]$$

$$= 52.4288 \text{ N/mm}^2$$

$$\sigma_B = 1.54 \left[21.6 \cos 14.47 + 68 \sin 14.47 \right]$$

$$= 6.1088 \text{ N/mm}^2$$

Similarly

$$\sigma_C = -5.8752 \ N/mm^2$$

$$\sigma_D = 23.4608 \ N/mm^2$$

$$\sigma_E = -96.3792 \ N/mm^2$$

$$\sigma_F = -93.2912 \ N/mm^2$$

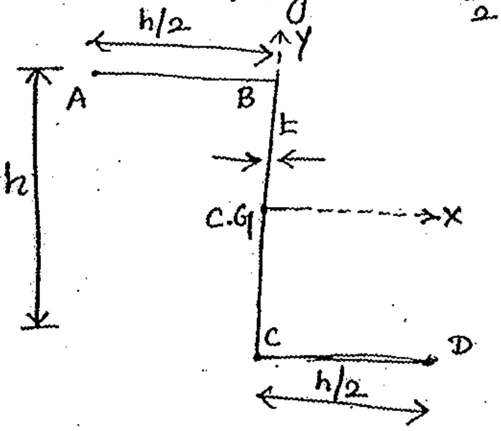
$$\sigma_G = 26.5488 \ N/mm^2$$

$$\sigma_H = 40.448 \ N/mm^2$$

$\therefore$ Maximum Bending stress is at F = 96.247 N/mm (compression

Problem :- 14

Determine the direct stress distribution in a thin walled z-section Produced by a Positive bending moment M_x. where as height of the section = h ; flange with = $\frac{h}{2}$ with thickness "t".



The section properties are

$$I_{xx} = \frac{th^3}{12} + 2\left[\frac{\frac{h}{2}\times t^3}{12} + \left(\frac{h}{2}t\right)\left(\frac{h}{2}\right)^2\right]$$

Since t is very small t^3 can be neglected

$$\therefore I_{xx} = \frac{th^3}{12} + \frac{th^3}{4} = \frac{th^3}{3}$$

$$I_{yy} = \frac{ht}{12} + 12\left[\frac{t\,(h/2)^3}{12} + \left(\frac{t \times h}{2}\right)\left(\frac{h}{4}\right)^2\right]$$

$$= 2\left[\frac{th^3}{96} + \frac{th^3}{32}\right] = 2\left[\frac{4\,th^3}{96}\right] = \frac{th^3}{12}$$

$$I_{xy} = \left(\frac{h}{2} \times t\right)\left(\frac{h}{4}\right)\left(\frac{-h}{2}\right) + \left(\frac{h}{2} \times t\right)\left(\frac{-h}{4}\right)\left(\frac{h}{2}\right)$$

$$= -\frac{th^3}{16} - \frac{th^3}{16} = -\frac{th^3}{8}$$

Given $M_x = M_x$; $M_y = 0$

WKT $$\sigma_z = \frac{\left(-M_y I_{xy} + M_x I_{yy}\right) y + \left(-M_x I_{xy} + M_y I_{xx}\right) x}{I_{xx} I_{yy} - I_{xy}^2}$$

Since $M_y = 0$.

$$\sigma_z = \frac{(M_x I_y)\, y - (M_x I_{xy})\, x}{I_x I_y - I_{xy}^2}$$

$$= \frac{\left(M_x \times \frac{th^3}{12}\right) y + \left(M_x \times \frac{-th^3}{8}\right) x}{\frac{th^3}{3} \times \frac{th^3}{12} - \left(\frac{th^3}{8}\right)^2} = \frac{\frac{M_x y}{12} \mp \frac{M_x x}{8}}{th^3 \times 7/576}$$

$$= \frac{576}{7} M_x \left[\frac{y}{12} \mp \frac{x}{8}\right] \times \frac{1}{th^3}$$

$$= \frac{576}{7} M_x \left[\frac{8y \mp 12x}{96}\right] \times \frac{1}{th^3}$$

$$\sigma_z = \frac{6M_x}{7th^3}\left[8y \mp 12x\right]$$

Location of Points,

Point	x	y
A	$-h/2$	$h/2$
B	0	$h/2$
C	0	$-h/2$
D	$h/2$	$-h/2$

$$\therefore \sigma_A = \frac{6M_x}{7th^3}\left[\frac{8h}{2} + \frac{12h}{2}\right] = +\frac{16M_x}{7th^2} = \frac{\overset{2.285}{\cancel{4.0744}}\,M_x}{th^2}$$

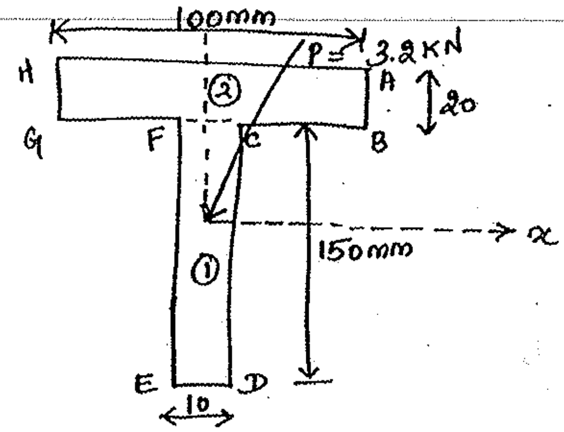
$$\sigma_B = \frac{6M_x}{7th^3}\left[\frac{8h}{2} + 0\right] = \frac{24M_x}{7th^2} = \frac{3.428\,M_x}{th^2}$$

$$\sigma_C = \frac{6M_x}{7th^3}\left[\frac{-8h}{2} + 0\right] = \frac{-24M_x}{7th^2} = -\frac{3.428\,M_x}{th^2}$$

$$\sigma_D = \frac{6M_x}{7th^3}\left[\frac{-8h}{2} \mp \frac{12h}{2}\right] = -\frac{16M_x}{7th^2} = \frac{\overset{-2.285}{\cancel{4.0744}}\,M_x}{th^2}$$

Problem.:- 15

A beam of "T" Section (Flange 100 mm x 20 mm) & web (150 x 10mm) is 2.5 m in length and is Simply Supported at the ends. It carries a load of 3.4 KN inclined at 20° to the vertical and Passing through the centroid of the Section. If $E = 200\ GN/m^2$ Find Maximum tensile Stress, ~~com~~ Maximum compressive Stress, Deflection due to the load & Position of Neutral axis.



$A_1 = 150 \times 10 = 1500 mm^2$ $\quad A_2 = 100 \times 20 = 2000 mm^2$

$x_1 = 45 + \frac{10}{2} = 50 mm$ $\quad x_2 = \frac{100}{2} = 50 mm$

$y_1 = \frac{150}{2} = 75 mm$ $\quad y_2 = 150 + \frac{20}{2} = 160 mm$

$\therefore \bar{x} = 50 mm$ (symmetric about y axis)

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{2} = \frac{(1500)(75) + (2000)(160)}{1500 + 2000}$$

$\bar{y} = 123.571$ mm

$$I_{xx} = \frac{b_1 d_1^3}{12} + A_1(\bar{y} - y_1)^2 + \frac{b_2 d_2^3}{12} + A_2(\bar{y} - y_2)^2$$

$$= \frac{10 \times 150^3}{12} + (1500)(123.571 - 75)^2 + \frac{20 \times 100^3}{12} + (2000)(123.57 - 160)^2$$

$I_{xx} = 9072023.81 mm^4$

$$I_{yy} = \frac{d_1 b_1^3}{12} + A_1(\cancel{\bar{x} - x_1})^2 + \frac{d_2 b_2^3}{12} + A_2(\cancel{\bar{x} - x_2})^2$$

$$= \frac{150 \times 10^3}{12} + \frac{100 \times 20^3}{12}$$

$I_{yy} = 1699166.667 mm^4$

$I_{xy} = 0$ (Since symmetric about y-axis)

$$M = \frac{Wl}{4} = \frac{3.2 \times (2.5 \times 10^3)}{4} = 2000 \text{ kN-mm}$$

$$M_x = M \cos 20^\circ = 1879.385 \text{ kN-mm}$$

$$M_y = -M \sin 20^\circ = -684.040 \text{ kN-mm}$$

$$\sigma_z = \frac{(-M_y I_{xy} + M_x I_{yy})y + (-M_x I_{xy} + M_y I_{xx})x}{I_{xx} I_{yy} - I_{xy}^2}$$

Since $I_{xy} = 0$.

$$\sigma_z = \frac{M_x}{I_{xx}} y + \frac{M_y}{I_{yy}} x$$

$$= \frac{1879.385}{9072023.81} y + \frac{(-684.040)}{1679166.667} x$$

$$\sigma_z = 2.071627 \times 10^{-4} y \mp 4.073687 \times 10^{-4} x.$$

Location of points with respect to centroid.

Point	x	y
A	50	46.429
B	50	26.426
C	5	26.426
D	5	-123.571
E	-5	~~26.426~~ -123.571
F	-5	26.426
G	-50	26.426
H	-50	46.429

$$\sigma_A = (2.0716 \times 10^{-4})(46.429) \mp (4.0736 \times 10^{-4})(50)$$

$$= \quad \text{kN/mm}^2.$$

$$\sigma_B = (2.0716 \times 10^{-4})(26.426) + (4.0736 \times 10^{-4})(50)$$

$$= \quad \text{kN/mm}^2.$$

$||^{ly}$ $\sigma_C =$ KN/mm^2

$\sigma_D =$ KN/mm^2

$\sigma_E =$ KN/mm^2

$\sigma_F =$ KN/mm^2

$\sigma_G =$ KN/mm^2

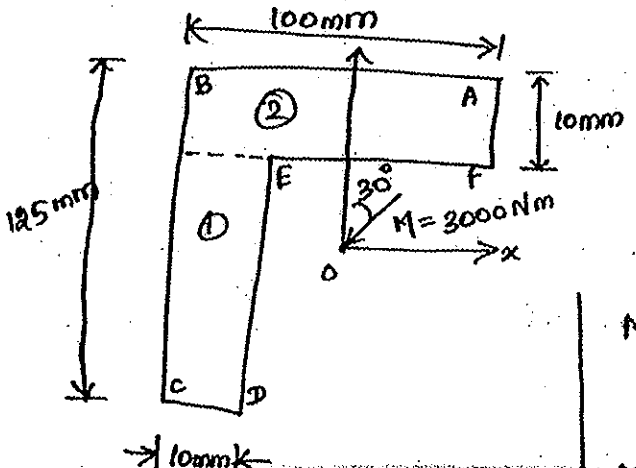
$\sigma_H =$ KN/mm^2

Maximum tensile stress is $\sigma_{tensile} =$ KN/mm^2

Maximum compressive stress is $\sigma_{comp} =$ KN/mm^2

Problem:. 16

A bending Moment of 3000 N-m is applied to the Section shown figure at an angle of 30° to the Vertical y axis. If the Sense the bending moment is such that its component M_x & M_y both oduces tension in the positive x-y quadrant. Calculate the maximum lirect stress on the Section.



$M_x = M\cos 30° = 3000 \times \cos 30°$

$= 2598.076$ N-m

$M_y = -M\sin 30° = -3000 \sin 30°$

$= -1500$ Nm.

$A_1 = 10 \times 115$
$= 1150\ mm^2$

$A_2 = 100 \times 10 = 1000\ mm^2$

$x_1 = \frac{10}{2} = 5mm$

$x_2 = \frac{100}{2} = 50mm$

$y_1 = \frac{115}{2} = 57.5mm$

$y_2 = 115 + \frac{10}{2} = 120\ mm$

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1+A_2} = \frac{(1150)(5)+(1000)(50)}{1150+1000}$$

$$\bar{x} = 25.93023 \text{ mm}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1+A_2} = \frac{(1150)(57.5)+(1000)(120)}{1150+1000}$$

$$\bar{y} = 86.56976 \text{ mm}$$

$$I_{xx} = \frac{b_1d_1^3}{12} + A_1(\bar{y}-y_1)^2 + \frac{b_2d_2^3}{12} + A_2(\bar{y}-y_2)^2$$

$$= \frac{10\times115^3}{12} + (1150)(86.56976-57.5) + \frac{100\times10^3}{12} + 1000(86.56976-120)^2$$

$$I_{xx} = 3365118.702 \text{ mm}^4$$

$$I_{yy} = \frac{d_1b_1^3}{12} + A_1(\bar{x}-x_1)^2 + \frac{d_2b_2^3}{12} + A_2(\bar{x}-x_2)^2$$

$$= \frac{115\times10^3}{12} + (1150)(25.93023-5)^2 + \frac{10\times100^3}{12} + (1000)(25.93023-50)^2$$

$$I_{yy} = 1926056.202 \text{ mm}^4$$

$$I_{xy} = \Sigma AXY = A_1(\bar{x}-x_1)(\bar{y}-y_1) + A_2(\bar{x}-x_2)(\bar{y}-y_2)$$

$$= 1150(25.93023-5)(86.56976-57.5) + 1000(25.93023-50)(86.56976-120)$$

$$I_{xy} = 1504360.465 \text{ mm}^4$$

$$\sigma_z = \frac{[-M_yI_{xy} + M_xI_{yy}]y + [-M_xI_{xy} + M_yI_{xx}]x}{I_{xx}I_{yy} - I_{xy}^2}$$

Jb

$$= \frac{\left[(+1500\times10^{3}\times 1504360.465)+(2598.076\times10^{3}\times 1926056.202)\right]y + \left[(2598.076\times10^{3}\times 1504360.465)+(1500\times10^{3}\times 3365118.702)\right]x}{(3365118.702)(1926056.202)-(1504360.465)^{2}}$$

$= \quad y + \quad x.$

Location of Points with respect to the centroid

Point	x	y
A	74.0697	38.4302
B	-25.93023	38.43024
C	-25.93023	-86.56976
D	-15.93023	-86.56976
E	-15.93023	28.43024
F	74.06977	28.43024

$\sigma_A = (\quad)(38.4302) + (\quad)(74.0697)$

$= \quad N/mm^2.$

$y \quad \sigma_B = \quad N/mm^2$

$\sigma_C = \quad N/mm^2$

$\sigma_D = \quad N/mm^2$

$\sigma_E = \quad N/mm^2$

$\sigma_F = \quad N/mm^2.$

Maximum Bending Stress occurs at $\quad = \quad N/mm^2$ (Compression)